



# **Waikato Regional Transport Model**

## **Four Step Model Trip Distribution**

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### **Technical Note 33**

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December 2014

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## Four Step Model Trip Distribution

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### Technical Note 33

### Quality Assurance Statement

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## Table of Contents

1.	Purpose .....	1
2.	Introduction .....	2
2.1	Trip Distribution Model Form .....	2
2.2	The Distribution Factor .....	2
3.	Calibrating the Generalised Cost Coefficients .....	3
4.	Calibrating the Distribution Functions .....	7
5.	Comparison of 3 Step and 4 Step Modelled Trip Lengths .....	25

## 1. Purpose

The purpose of this note is to summarise the trip distribution methodology adopted for the study as reported in technical note 27 of the Three Step Model, and to report on the calibration and validation of the distribution functions for the Four Step Model.

## 2. Introduction

### 2.1 Trip Distribution Model Form

The gravity model form chosen for this model was:

$$T_{ij} = P_i K_i A_j L_j f(c_{ij})$$

Subject to the double constraints of:

$$i = \frac{P_i}{\sum_j T_{ij}}$$

$$L_j = \frac{A_j}{\sum_i T_{ij}}$$

Where:

$T_{ij}$	=	Trips between zones i and j
$P_i$	=	Productions at zone i
$A_j$	=	Attractions at zone j
$f(c_{ij})$	=	Some function of the impedance between zones i and j
$K_i L_j$	=	Balancing factors

The balancing factors are successively applied until there is convergence. The notation has been ignored in the rest of this note as it can be shown that  $\sum K_i$  and  $\sum L_j$  equal 1 over the balancing iterations.

### 2.2 The Distribution Factor

The distribution function can be approximated to an exponential line of the form:

$$f(c_{ij}) = e^{-\alpha C_{ij}}$$

Where:

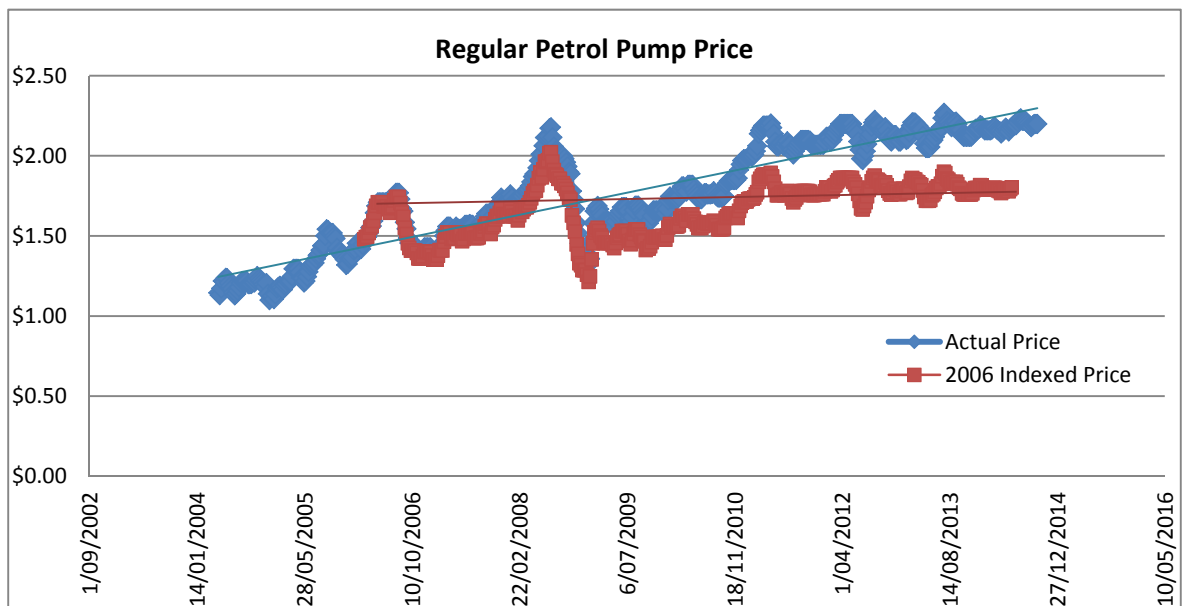
$f(c_{ij})$	=	function of cost of travel between zone i and zone j
$C_{ij}$	=	generalised cost of travel between zones i and j and is usually of the form $\beta t_{ij} + \gamma d_{ij} + \text{tolls} + \text{parking charges}$
$t_{ij}$	=	time to travel between zone i and zone j (minutes)
$d_{ij}$	=	distance between zone i and zone j (kilometres)
$\beta$	=	generalised cost of travel per minute
$\gamma$	=	generalised cost of travel per kilometre
$\alpha$	=	distribution model exponent to be calibrated

### 3. Calibrating the Generalised Cost Coefficients

The first step in the calibration is to evaluate the generalised cost coefficients,  $\beta$  and  $\gamma$ , for each purpose and time period. The time cost coefficient has been established using the values published in Table A4.1 of the New Zealand Transport Agency’s (NZTA) Economic Evaluation Manual (EEM).

Only part of the running costs for cars is considered to be a perceived cost. It is assumed that most car drivers, when deciding to make a trip by car, consider as their vehicle operating costs only the cost of fuel. This is consistent with the guidelines for private vehicle operating costs in “Travel Behaviour Change Evaluation Procedures” prepared for Transfund NZ (now part of NZTA) in December 2004 by Maunsells Australia Pty Ltd.

Data from the Ministry of Economic Development of fuel prices from 2004 to 2014 is shown on the diagram below. The trend line shows a price of \$2.16 for the first quarter of 2013.



**Graph 1: Fuel Prices**

Accordingly, the distance cost for cars has been calculated as 17.06 cents per kilometre, which corresponds to 10 kilometres per litre at \$1.71 per litre which has been indexed back to 2006 dollars as calculated from the Statistics New Zealand Consumer Price Index.

The resulting time and distance costs are detailed in **Table 1**.

Vehicle Class	Work Travel Purpose		Commuting to/from Work		Other Non-Work Travel Purposes		Distance Cost
	\$/Hour	Cents/mm	\$/Hour	Cents/mm	\$/Hour	Cents/mm	Cents/km
Car	23.85	39.75	7.80	13.00	6.90	11.50	17.06

**Table 1: Perceived Costs of Travel by Vehicle Class**

The non-work time values need to be factored by 1.15 to convert to perceived costs (Table A11.1 of EEM), and then by 1.19 to bring into mid 2008 values from 2002 (Table A12.2 of EEM).

The next stage was to determine the percentage of trips which fall into the broader work travel, commuting and non-work travel purposes by time of day for each modelled trip purpose, the average occupancy per vehicle, and then for assignment, the weighting of each purpose to form an overall generalised cost. It should be noted that Heavy Goods Vehicles have been dealt with separately as that model has been imported from the recent Christchurch Transport Model project.

A literature search to determine source the percentage breakdowns proved fruitless and in the absence of any such guidelines, the proportions in **Table 2** of trips classified as work, non-work, and commuting have been assumed for this analysis. These proportions have been used in numerous New Zealand applications since the early 1980s including public transport models in North Shore, Hamilton, Christchurch, Timaru and Dunedin.

	Period	Work	Non Work	Commuting
Home Based Work	All	0.0	0.0	1.0
Home Based Ed	All	0.0	0.0	1.0
Home Based Business	All	0.8	0.2	0.0
Home Based Shop	All	0.0	1.0	0.0
Home Based soc/rec	All	0.0	1.0	0.0
Home Based Other	All	0.2	0.8	0.0
Non Home Based	All	0.5	0.5	0.0

**Table 2: Proportion of Work and Non-work Trips**

**Table 3** includes the car occupancy values as calibrated from the Home Interview Survey. There is one anomaly in that Home Based Education is a trip to an education facility for the purpose of study. The car driver trip for the purpose of dropping a person off at school is classed as a Home Based Other trip.

For the purpose of calculating occupancies, the Home Based Work occupancy has been assumed for Home Based Education car drivers, and the Home Based Education passengers have been included in the home based other purpose.

	Morning Peak	Inter Peak	Evening Peak
Home To Work	1.07	1.05	-
Home To Ed	1.07	-	-
Home To Business	1.07	1.28	-
Home To Shop	1.24	1.14	1.19
Home To Soc/Rec	-	-	1.76
Home to Other	1.71	1.73	1.52
Non Home Based	1.43	1.38	1.41

	Morning Peak	Inter Peak	Evening Peak
Work To Home	-	1.05*	1.09
Ed To Home	-	-	1.09
Business To Home	-	1.28*	1.52
Shop To Home	-	1.14*	1.19
Soc/Rec To Home	-	-	-
Other To Home	1.20	1.73*	1.90
<b>Total</b>	<b>1.33</b>	<b>1.37</b>	<b>1.37</b>

**Table 3: Car Occupants by Time Period**

**Table 4** has the proportion of trips by purpose for the private purposes for each time period. Note that for the home-based (i.e. home-to and to-home) purposes only one of the directions is included towards the total<sup>1</sup>.

	Morning Peak	Inter Peak	Evening Peak
Home To Work	20.47	9.91	-
Home To Ed	23.02	-	-
Home To Business	1.73	5.32	-
Home To Shop	2.28	10.66	9.88*
Home To Soc/Rec	-	-	6.75
Home To Other	22.28	31.57	7.03
Non Home Based	22.25	42.53	27.44
Work To Home	-	9.91*	24.05
Ed To Home	-	-	4.79
Business To Home	-	5.32*	2.19
Shop To Home	-	10.66*	9.88
Soc/Rec To Home	-	-	-
Other To Home	7.98	31.57*	17.87
<b>Total</b>	<b>100.00</b>	<b>100.00</b>	<b>100.00</b>

**Table 4: Purpose Proportions by Time Period**

**Table 5** contains the resulting generalised costs coefficients for time and distance. The time costs are evaluated by multiplying the cost in Table 1 out by the proportions in Table 2, adjusting for perceived costs (1.15 factor for non-work related), scaling up to July 2008 dollars (1.19 factor for all) and then multiplying each by the occupancy rates in Table 3. Subsequently it represents the aggregated perceived time cost for all vehicle occupants.

<sup>1</sup> In Tables 3 and 4, trip purposes with an asterisk (\*) do not get included to produce the total, i.e. the trip purpose proportions are home-based rather than home-to and to-home.



	Morning Peak		Inter Peak		Evening Peak	
	Time(c)	Dist(c)	Time(c)	Dist(c)	Time(c)	Dist(c)
Home To Work	17.72	17.08	17.49	17.08	-	-
Home To Ed	17.59	17.08	-	-	-	-
Home To Business	40.92	17.08	48.94	17.08	-	-
Home To Shop	18.24	17.08	16.70	17.08	17.42	17.08
Home To Soc/Rec	-	-	-	-	25.88	17.08
Home To Other	33.76	17.08	30.97	17.08	33.94	17.08
Non Home Based	41.97	17.08	40.71	17.08	41.49	17.08
Work To Home	-	-	17.49	17.08	18.15	17.08
Ed To Home	-	-	-	-	18.15	17.08
Business To Home	-	-	48.97	17.08	58.13	17.08
Shop To Home	-	-	16.70	17.08	17.42	17.08
Soc/Rec To Home	-	-	-	-	-	-
Other To Home	23.86	17.08	30.97	17.08	35.41	17.08
<b>Total</b>	<b>27.60</b>	<b>17.08</b>	<b>33.21</b>	<b>17.08</b>	<b>30.07</b>	<b>17.08</b>

**Table 5: Generalised Costs by Time Period**

## 4. Calibrating the Distribution Functions

The surveyed trip matrices were derived from the Household Interview Survey data for each purpose and time period. The purposes used were:

Home to Work	Work to Home
Home to Education	Education to Home
Home to Business	Business to Home
Home to Shop	Shopping to Home
Home to soc/rec	Soc/rec to Home
Home to Other	Other to Home
Non home Based	

The TRACKS program DISCAL has been used to calculate the distribution function coefficients. DISCAL uses as inputs a time and distance matrix with corresponding generalised cost coefficients (as derived above) and a trip matrix. To start the process the time and distance matrices were derived from the all-day assignment of the surveyed trips matrix used to determine expansion factors (see Technical note 4).

The calibration process involves inverting the gravity model so that it is expressed in terms of the distribution function:

$$f(C_{ij}) = T_{ij} / P_i A_j$$

The function value is calculated for each origin/destination pair, and allocated to a cost band k. The final value of the function in each cost band is the weighted average of the individual cells in that band.

The natural log of this average is then calculated and plotted against cost to calibrate the alpha value for use in the negative exponential function form.

It should be noted that the cost function can also be a function of travel time between zones rather than cost, and is the form that was adopted for the vehicle driver (three step) models and reported in Technical Note 27. The person models (four step) have used generalised cost because of the need to bring in the public transport travel cost components.

The first stage in the process was to calibrate the alpha exponent using the surveyed matrices and the period time and distance matrices derived from assignment of the surveyed matrices. The results from this analysis for each purpose and time period are shown in the Figures below, including the number of trips in each of the matrices and the regression r<sup>2</sup> values.

The results of the calibration for the person model (cost based) are included in Figures 1a-3c respectively. Where a purpose is not included, the number of observations for that purpose and time period is insufficient for calibration purposes. Note that although a 4-step evening peak model has not been developed, the calibrated alpha exponents have been presented here in Figure 3 for completeness sake.

Please see Table One of Technical Note 10 for a breakdown of the total number of expanded HIS trips for each purpose. This and the relevant discussion in the corresponding section of Technical Note 10 provides some perspective regarding the level of sampling for each.

The goal is to produce R-squared values of greater than 0.8 where possible, which implies that the calibrated function coefficient explains at least 80% of the surveyed data. In some cases this has not been possible due to the low number of sampled and expanded households at a trip purpose by period level.

Note that the total expanded trips are included on each plot, with the number of sampled trips being approximately 1% of these values based on an average expansion factor of approx 100.

In general terms the standard deviations are higher in the 4-step model as there is more variability in the trip lengths when all modes are included in the analysis. Subsequently it may be more difficult to get as close a fit in terms of an R-squared statistic. For this reason the 4-step model R-squared values are generally lower than the 3-step model equivalents.

The second stage is to apply the gravity model using modelled trip ends, and confirm that the modelled and observed trip length frequencies still match. There is an iterative process between assignment and distribution that will eventually converge. Throughout this process the alpha values published in Figure 1 and Figure 2 are successively ‘tuned’ such that the modelled trip lengths match the HIS trip lengths as closely as possible. As such the alpha values in Figure 1 and Figure 2 are treated as a ‘starting point’ and the final validated alpha values, which are the outcome of this process are reported in Table 6.

Figure 3a through Figure 4d show the results of using the calibrated functions in the four-step morning peak and interpeak models. These figures show the trip length frequency plots for time and distance for each statistically significant trip purpose. For each purpose, both the modelled and HIS frequency plots have been provided along with statistical measures for both.

Again, in instances where the number of observations for a purpose was too small, plots have not been provided. A full table of HIS and modelled averages and standard deviations has been included here to summarise the results published in Figure 3a through Figure 4d as **Table 6** and summarised in **Table 7**.

Note that trip chaining has not been taken into account in this analysis. The analysis focuses on trip legs only. This is consistent with all interpretation and analysis of the WRTM Household Interview Survey data.

	Morning Peak	Inter Peak
Home To Work	.0033	.0047
Home To Ed	.0080	.0256
Home To Business	.0040	.0018
Home To Shop	.0060	.0042
Home To Soc/rec	-	-
Home To Other	.0040	.0028

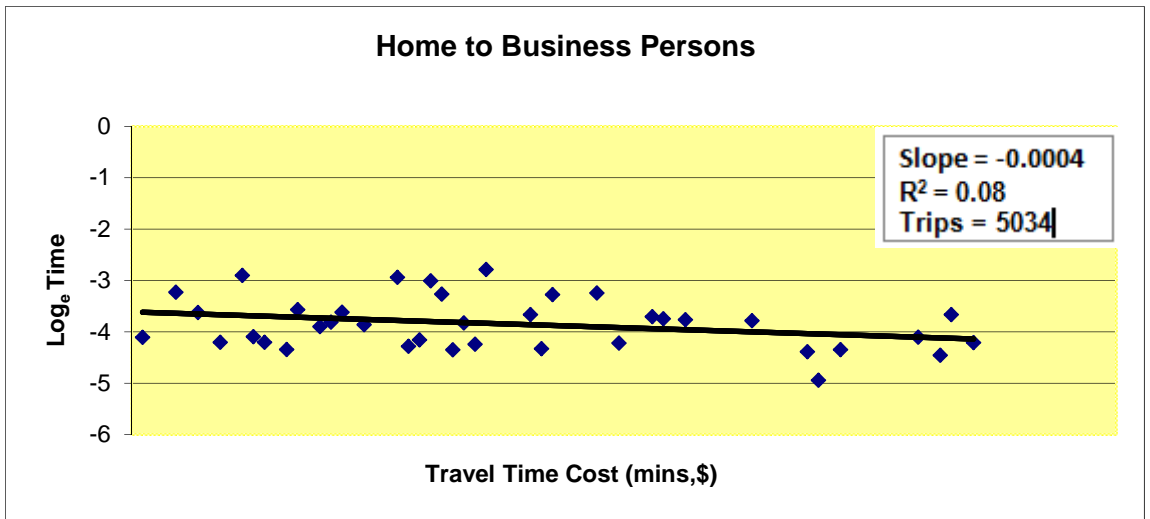
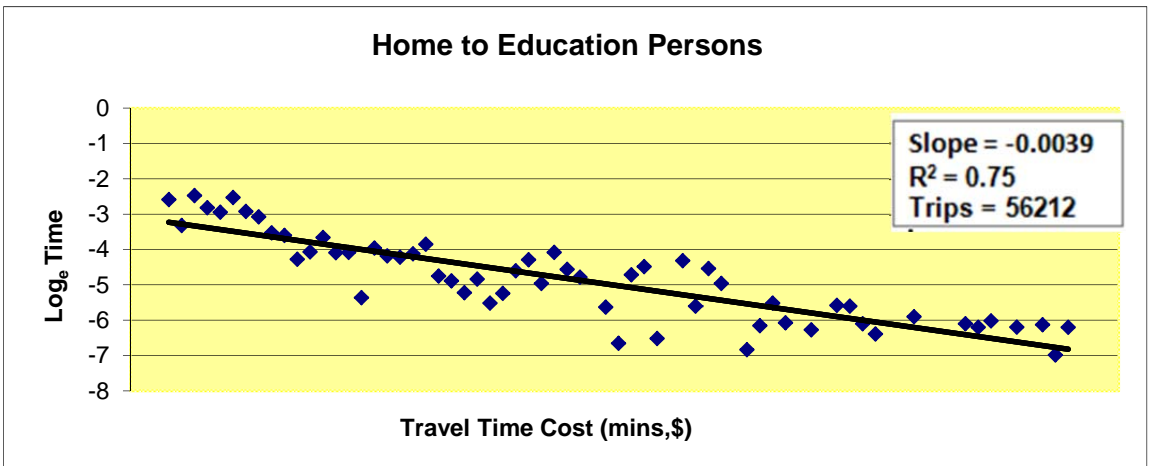
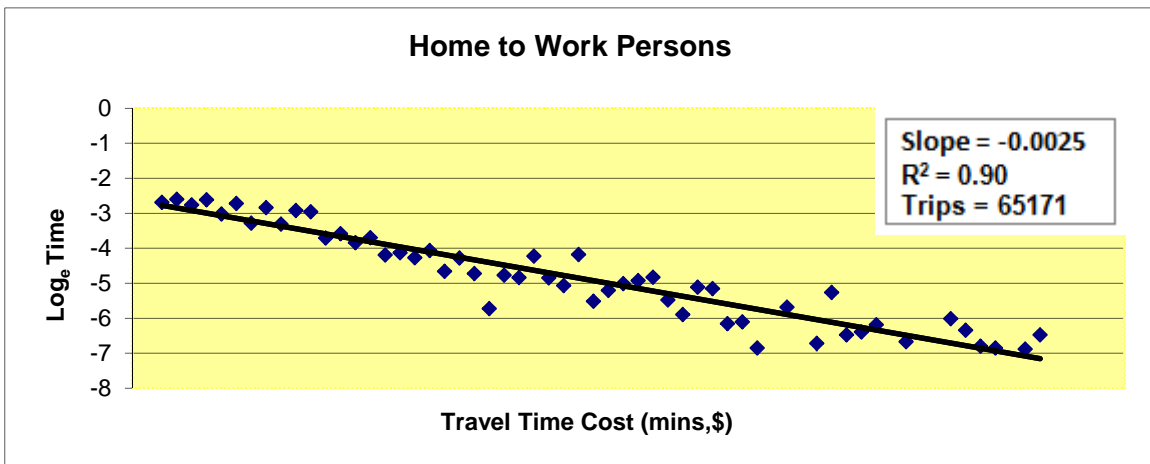
	Morning Peak	Inter Peak
Non Home Based	.0021	.0021
Work To Home	-	.0032
Ed To Home	-	.0085
Business To Home	-	.0016
Shop To Home	-	.0073
Soc/rec To Home	-	-
Other To Home	.0060	.0035

**Table 6: Validated Distribution Model Coefficients**

Period	Purpose	Figure	Trip Cost (\$)			
			HIS		Model	
			Mean	SD	Mean	SD
Morning Peak	Home to Work	4A	4.23	4.80	4.22	3.96
	Home to Education	4A	2.41	2.64	2.41	2.28
	Home to Business	4A	7.97	10.03	7.94	8.58
	Home to Shop	4B	3.34	5.08	3.37	3.57
	Home to Social/Rec	-	-	-	-	-
	Home to Other	4B	4.18	4.86	4.15	3.64
	Non Home Based	4C	5.42	8.61	5.85	5.86
	Other to Home	4C	2.71	4.01	2.70	2.37
	Work to Home	-	-	-	-	-
	Education to Home	-	-	-	-	-
	Business to Home	-	-	-	-	-
	Shop to Home	-	-	-	-	-
	Soc/Rec to Home	-	-	-	-	-
Interpeak	Home to Work	5A	4.06	6.38	4.07	4.09
	Home to Education	5A	2.78	3.41	2.79	3.72
	Home to Business	5A	10.15	13.45	10.22	10.80
	Home to Shop	5B	4.15	7.00	4.16	4.65
	Home to Social/Rec	-	-	-	-	-
	Home to Other	5B	5.33	7.84	5.27	4.97
	Non Home Based	5B	5.58	10.77	5.56	5.35
	Other to Home	5C	4.57	6.66	4.57	4.38
	Work to Home	5C	4.46	6.64	4.48	4.20
	Education to Home	5C	2.51	2.90	2.50	2.46
	Business to Home	5D	9.60	10.09	9.45	8.95
	Shop to Home	5D	3.40	4.70	3.38	3.85
	Soc/Rec to Home	-	-	-	-	-

**Table 7: Comparison of HIS and Modelled Trip Length**

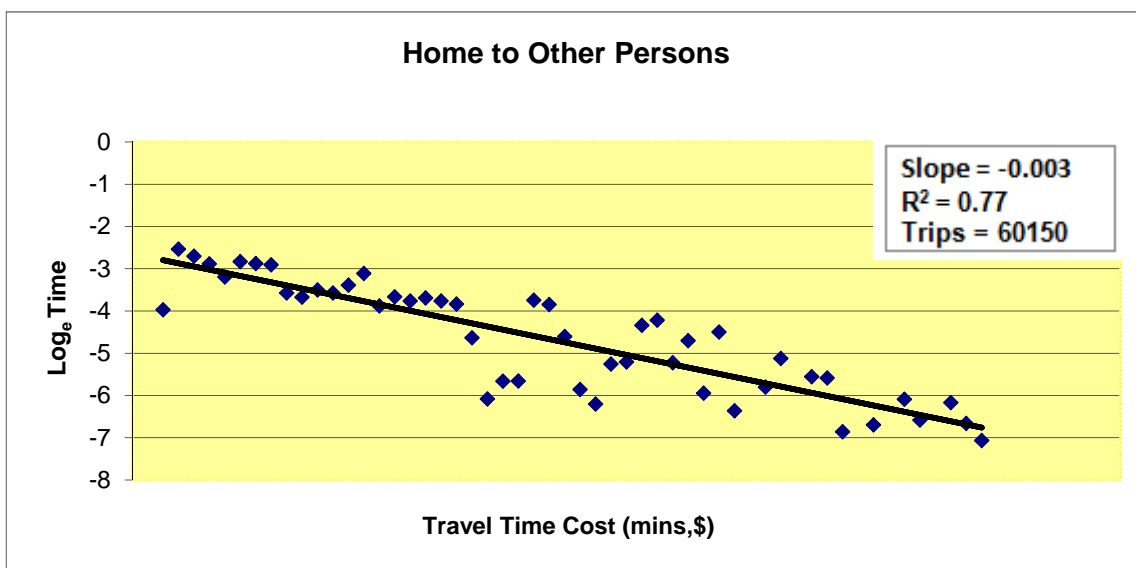
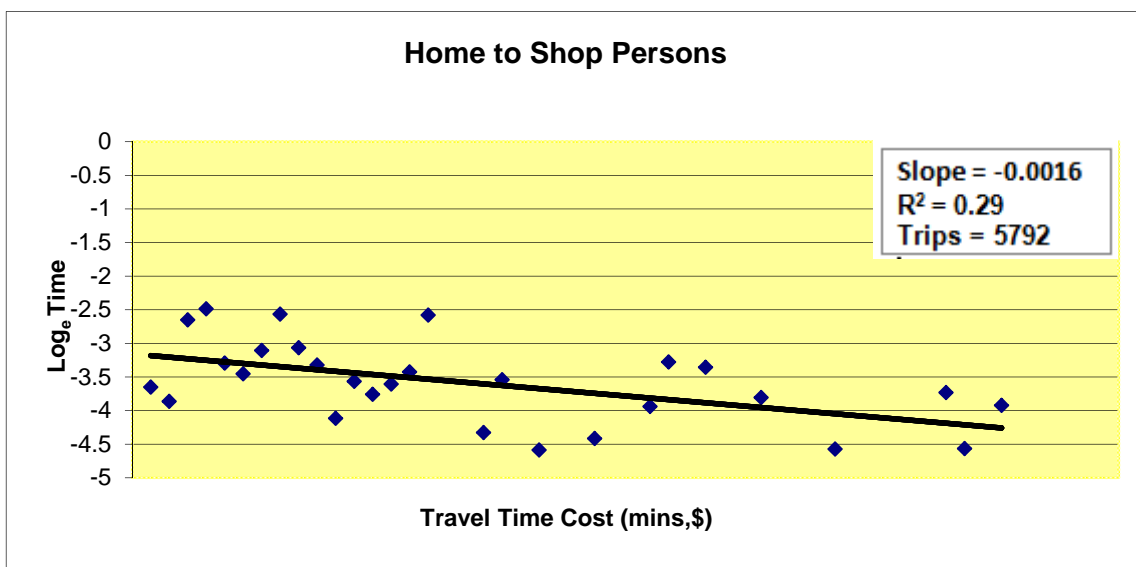
It is interesting to note that there are differences between trip lengths for 'home to' and 'to home' trip purposes. In all cases any such variability is not significant due to the high standard deviations relative to the means. This implies that there is a wide spread of trip lengths for each purpose and period, and therefore any minor differences between the 'home to' and 'to home' are negligible.

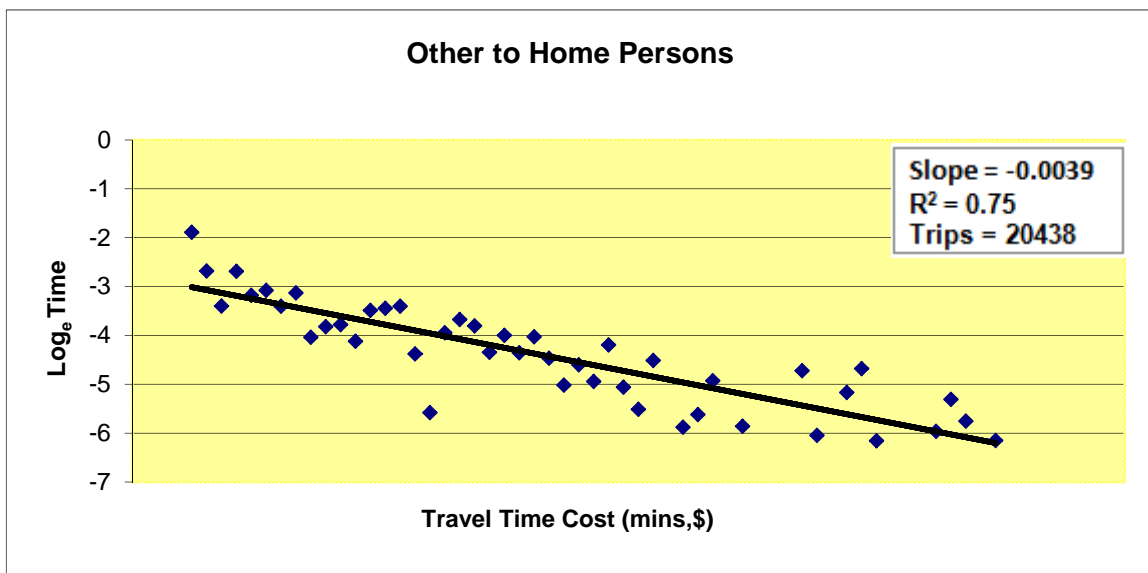
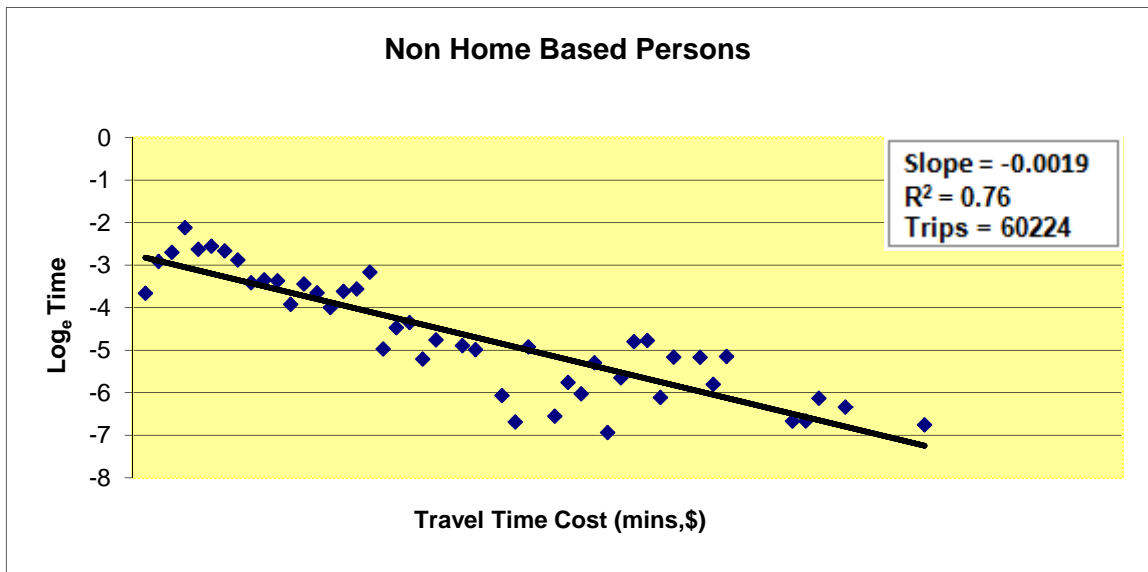


Distribution Functions (HTW/HTE/HTB)  
Morning Peak Person Trips



1a



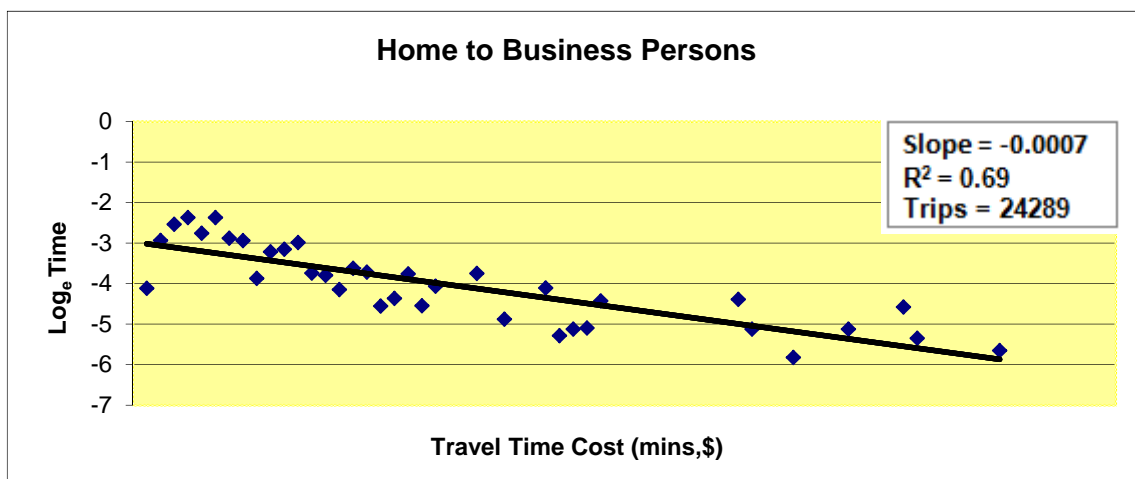
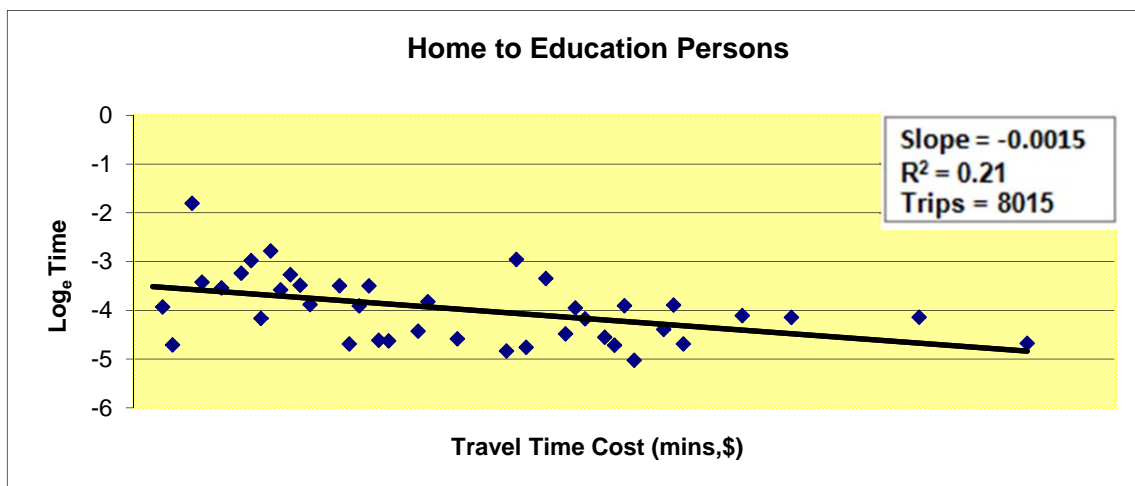
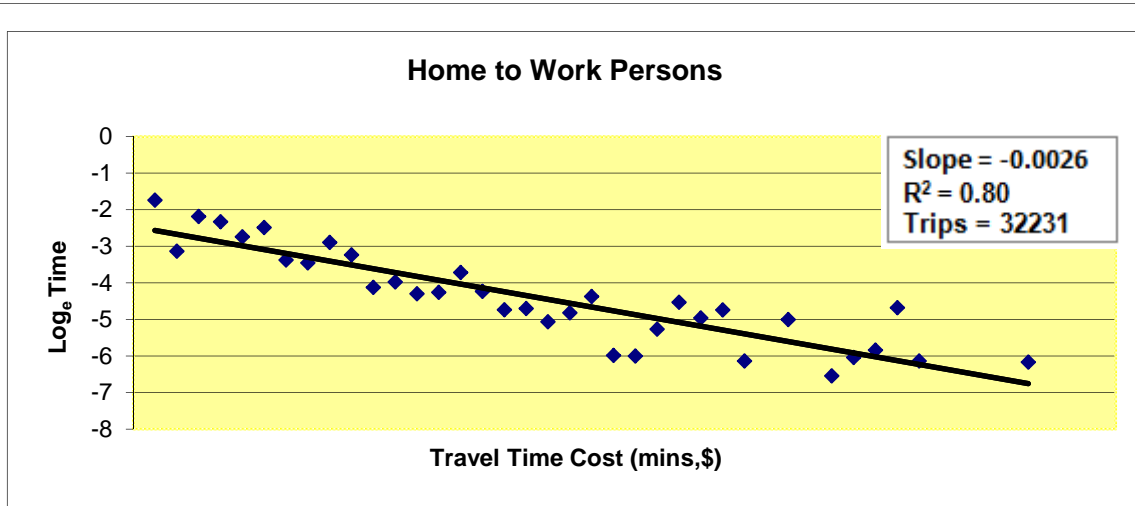


Distribution Functions (NHB/OTH)  
Morning Peak Person Trips



1c

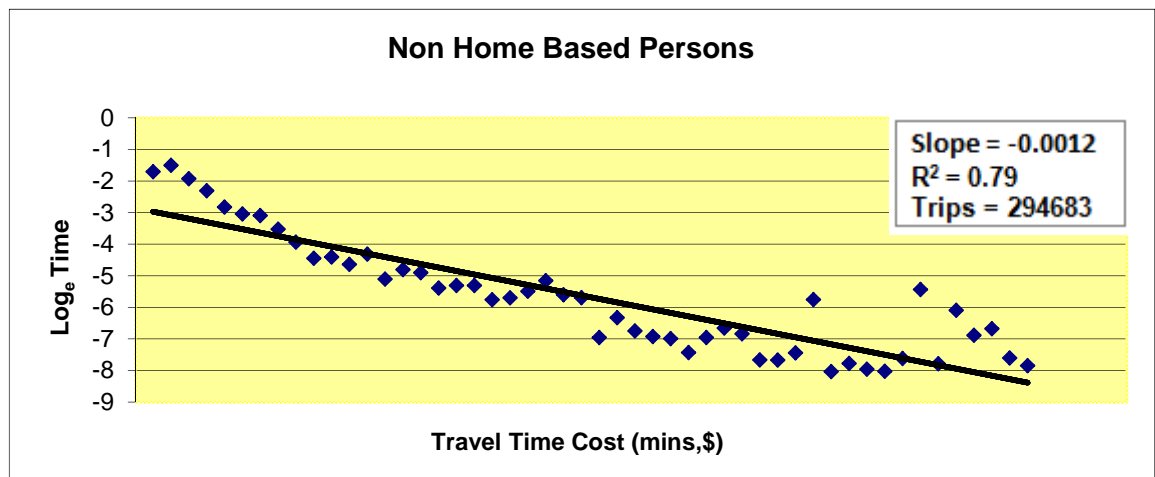
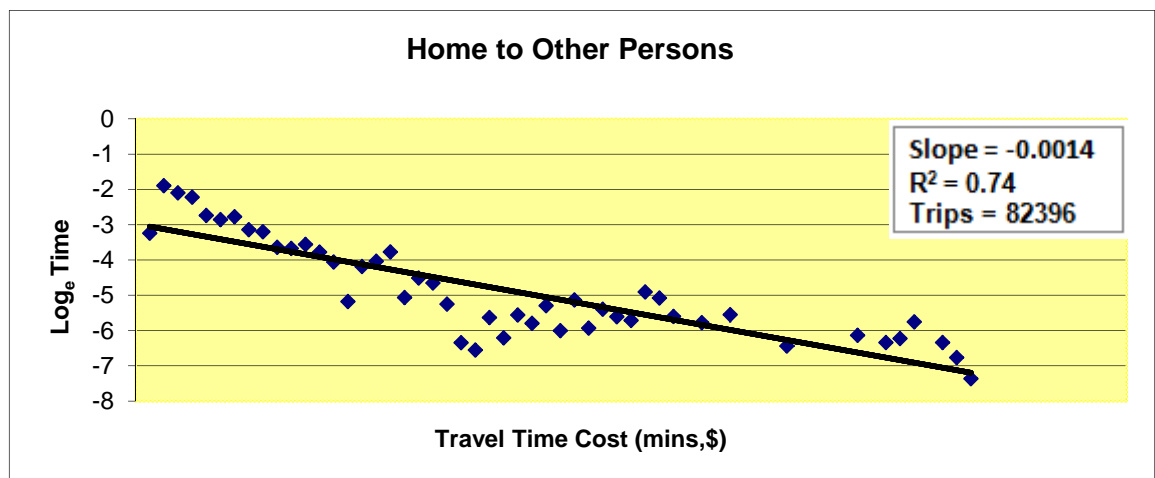
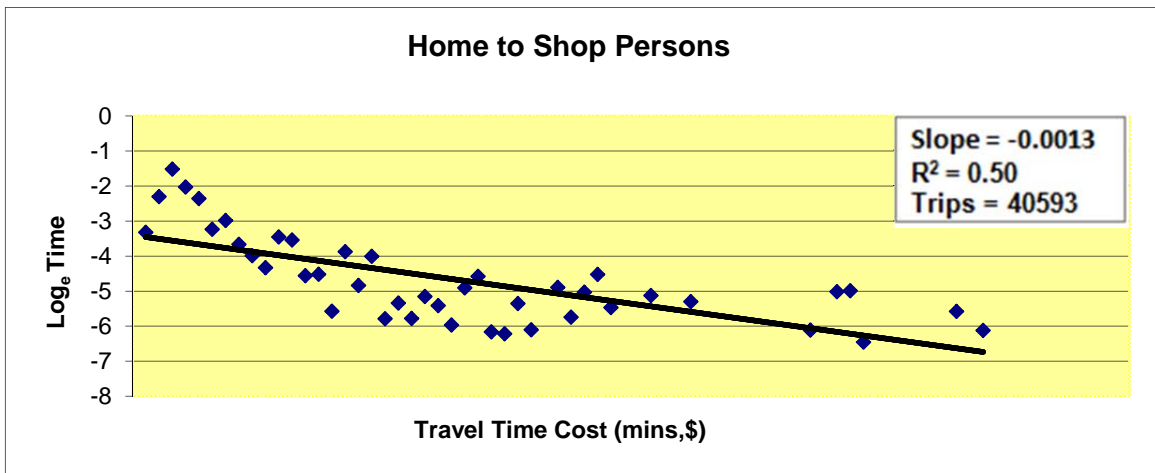




Distribution Functions (HTW/HTE/HTB)  
Inter Peak Person Trips



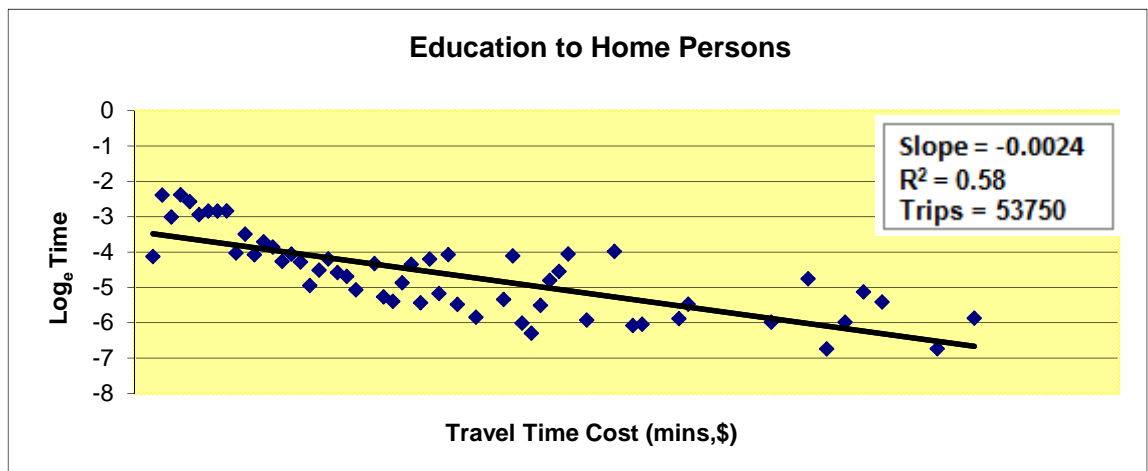
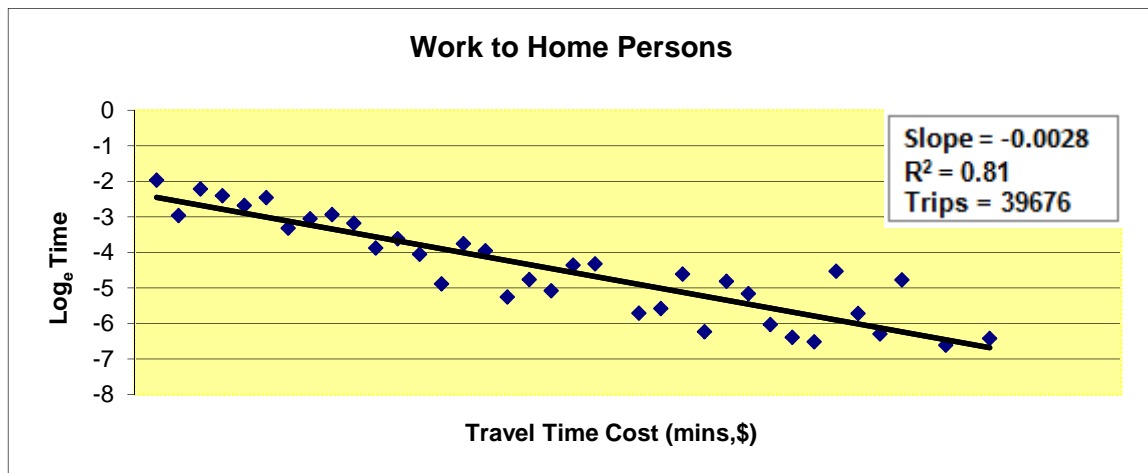
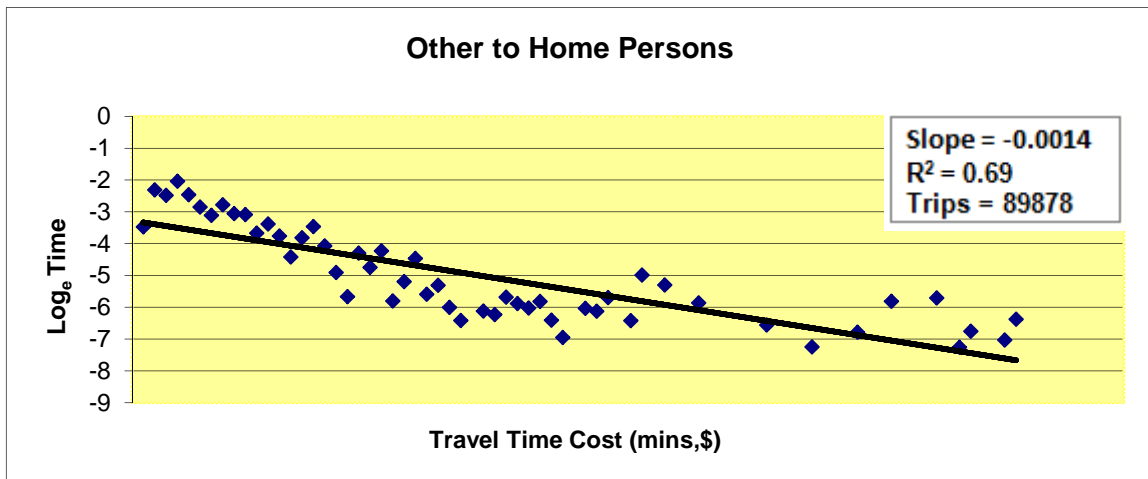
2a



Distribution Functions (HTS/HTO/NHB)  
Inter Peak Person Trips



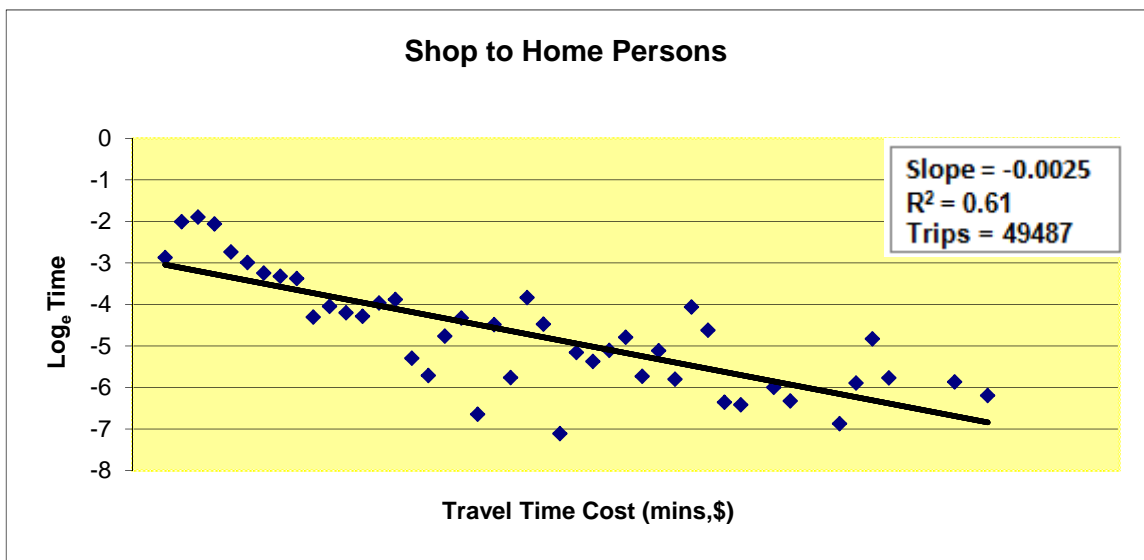
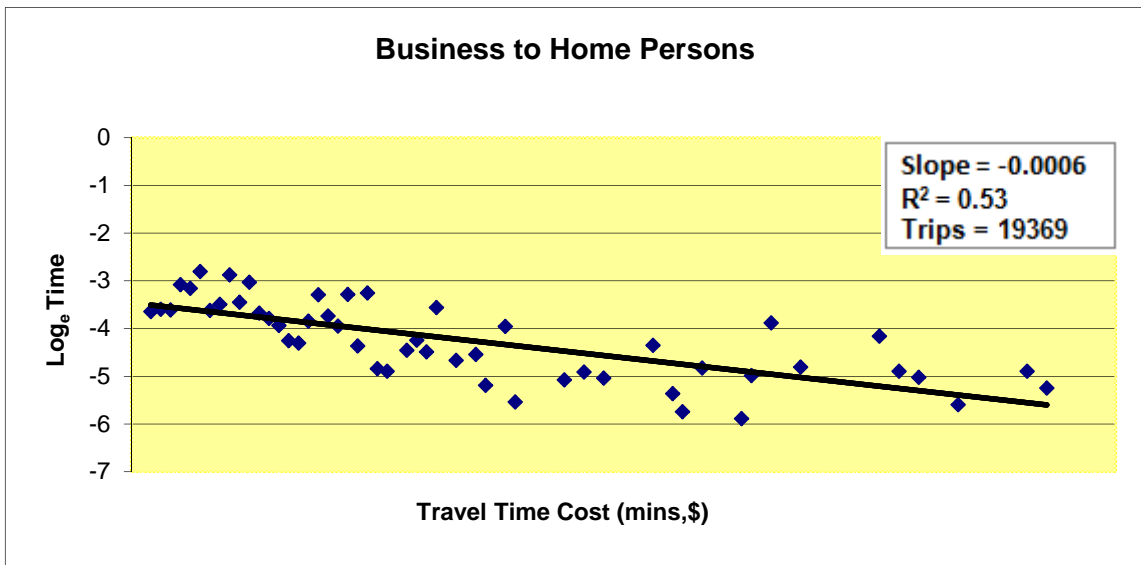
2b



Distribution Functions (OTH/WTH/ETH)  
Inter Peak Person Trips



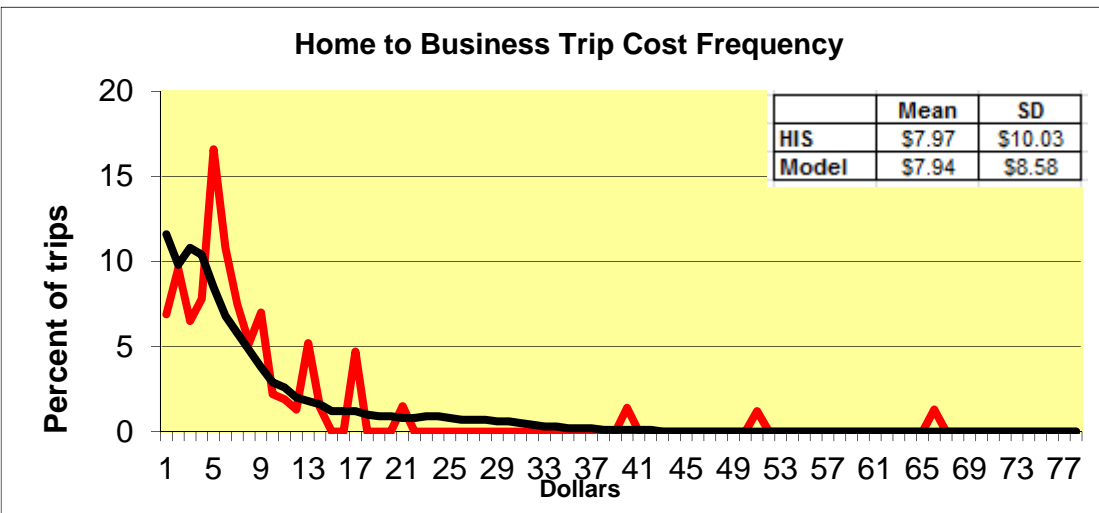
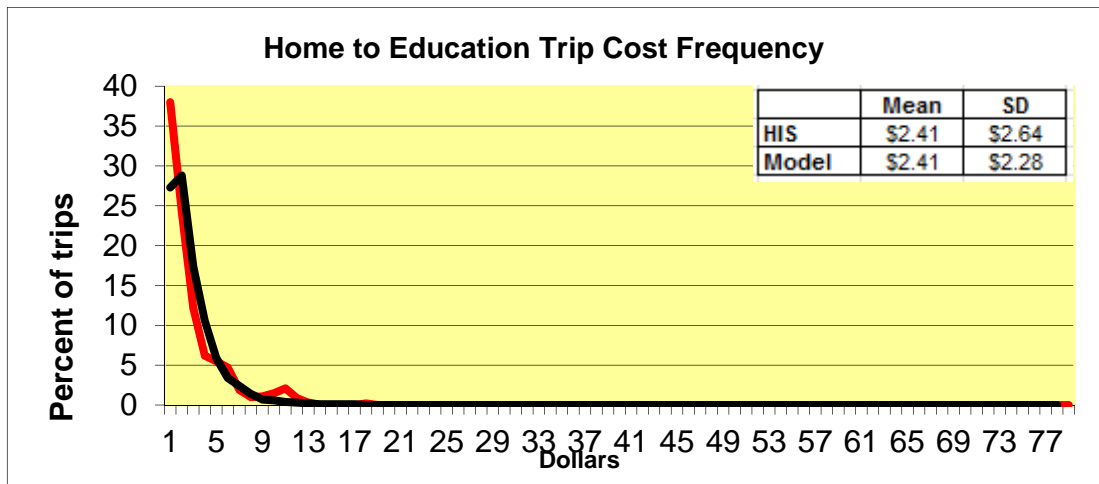
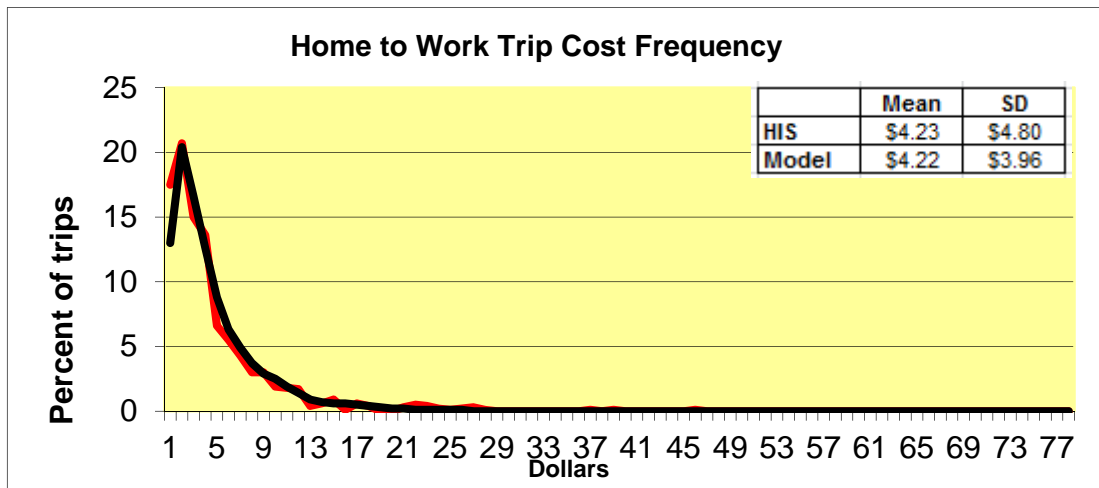
2c



Distribution Functions (BTH/STH)  
Inter Peak Person Trips



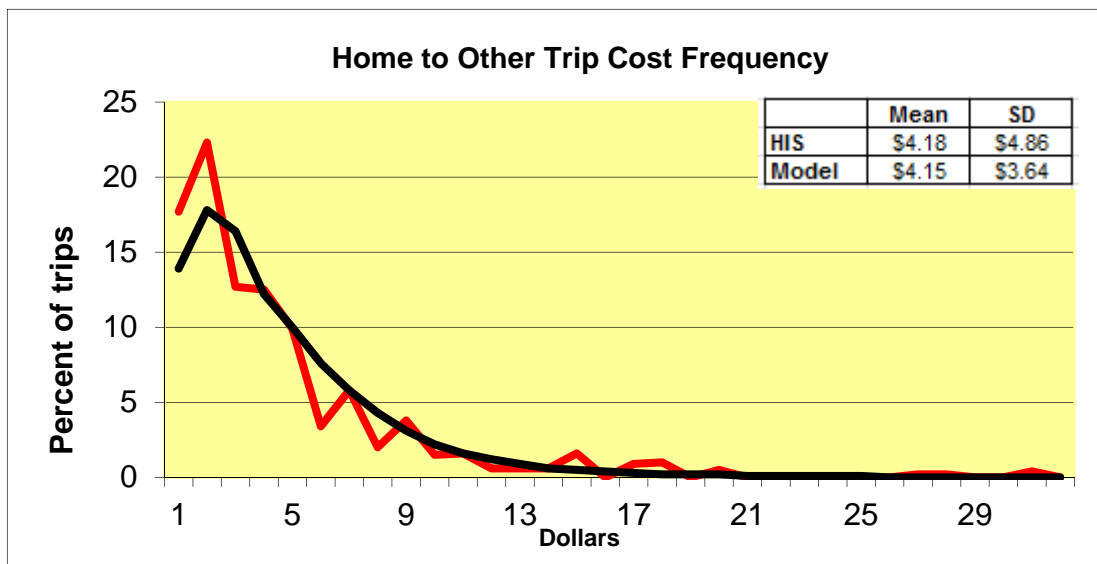
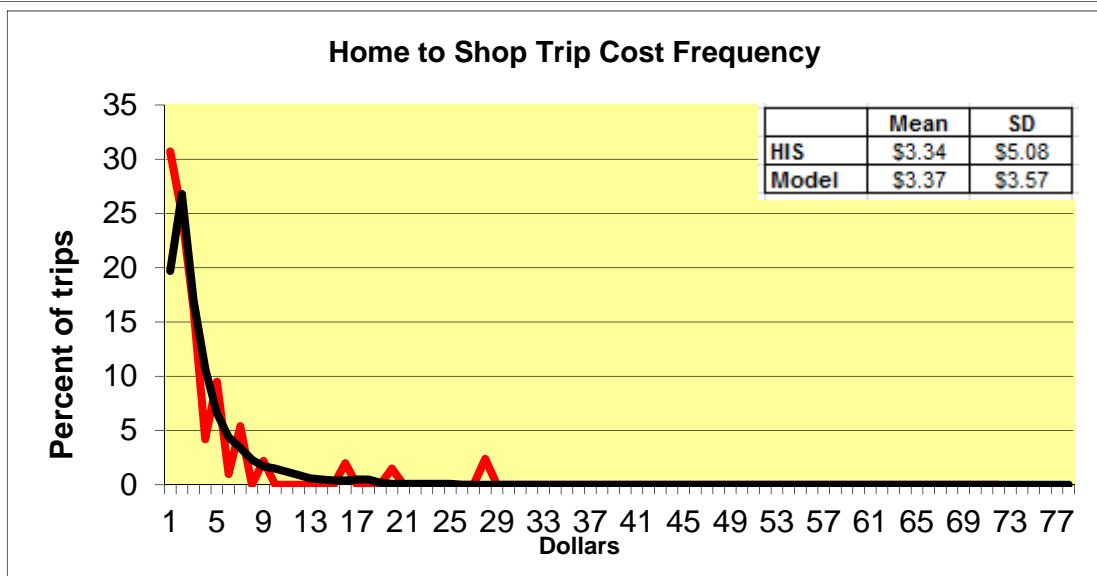
2d



Trip Cost Frequency Plots (HTW/HTE/HTB)  
 Morning Peak Person Trips



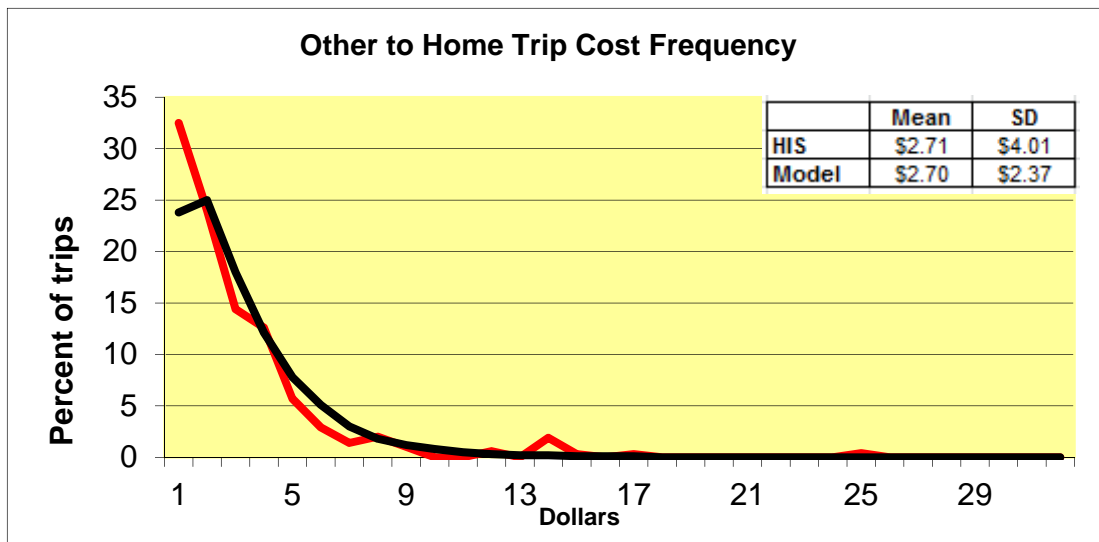
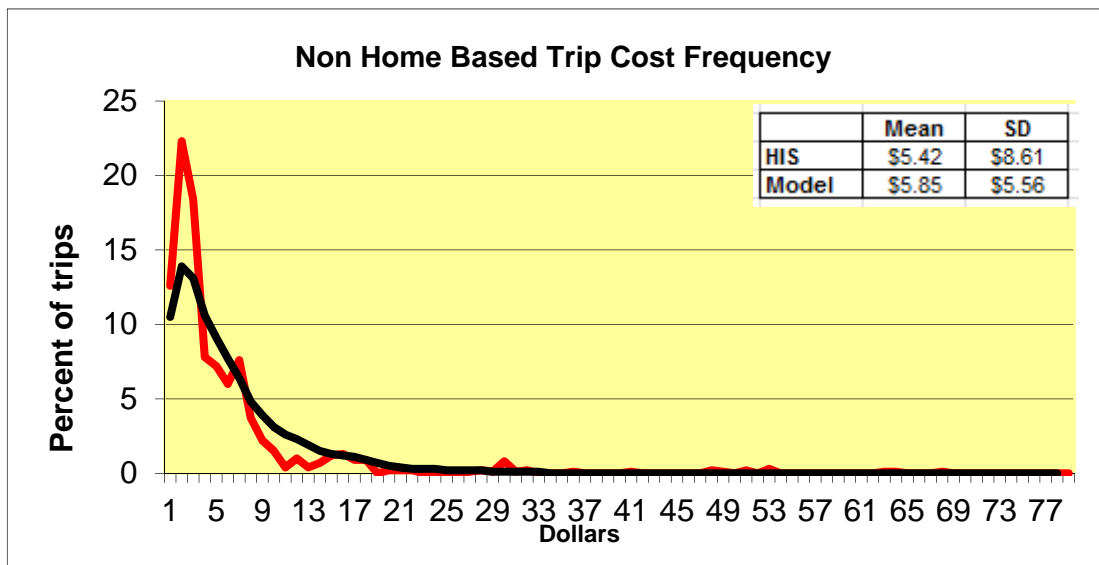
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Trip Cost Frequency Plots (HTS/HTO)  
 Morning Peak Person Trips



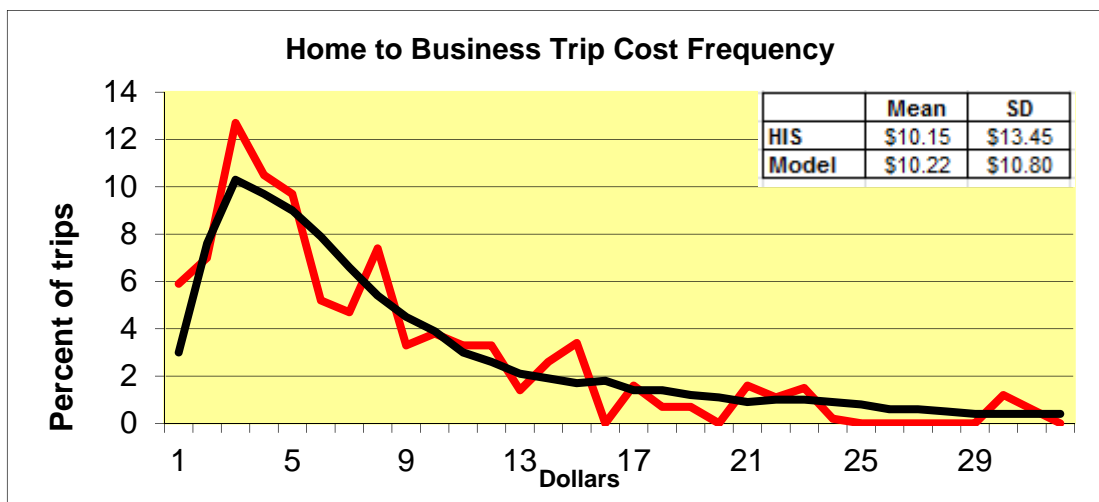
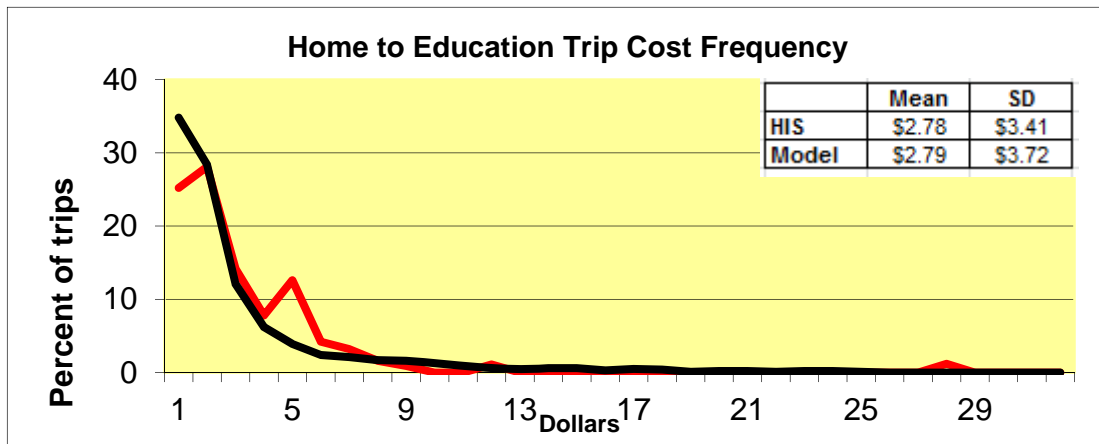
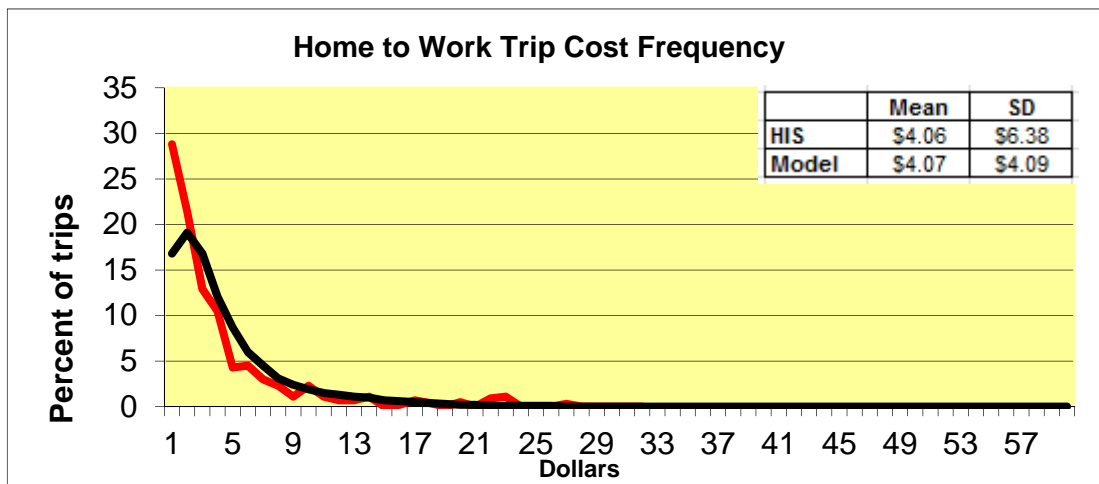
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Trip Cost Frequency Plots (NHB/OTH/WTH)  
 Morning Peak Person Trips



3c

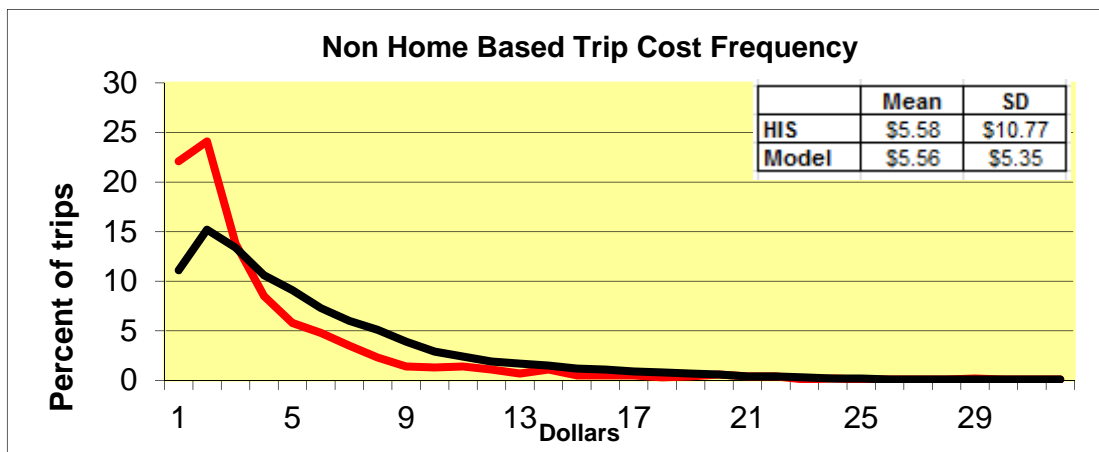
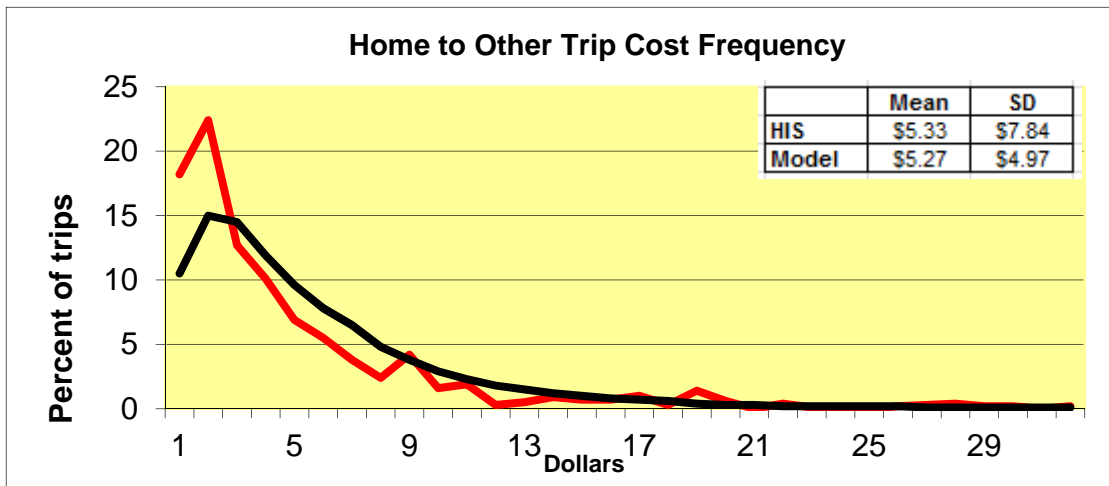
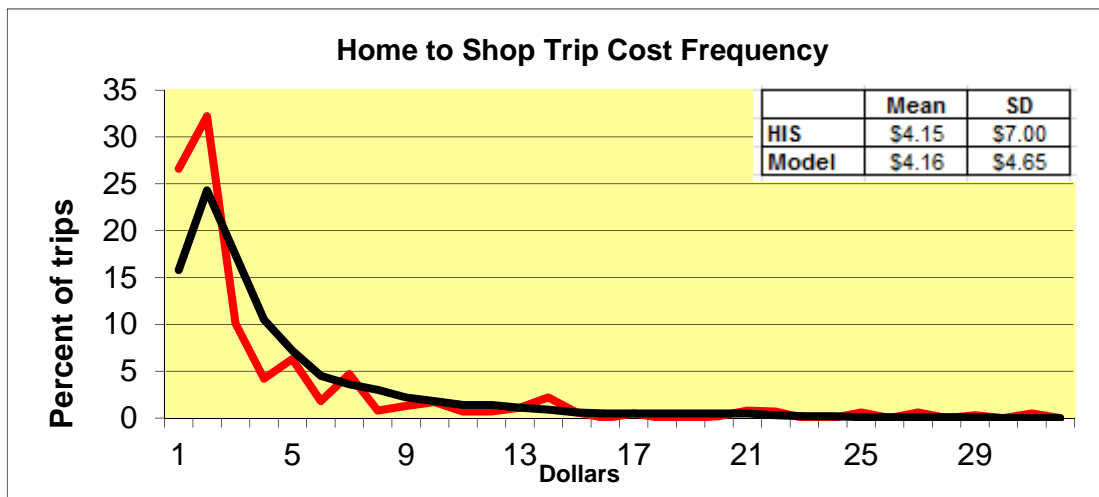


Trip Cost Frequency Plots (HTW/HTE/HTB)  
 Inter Peak Person Trips



4a

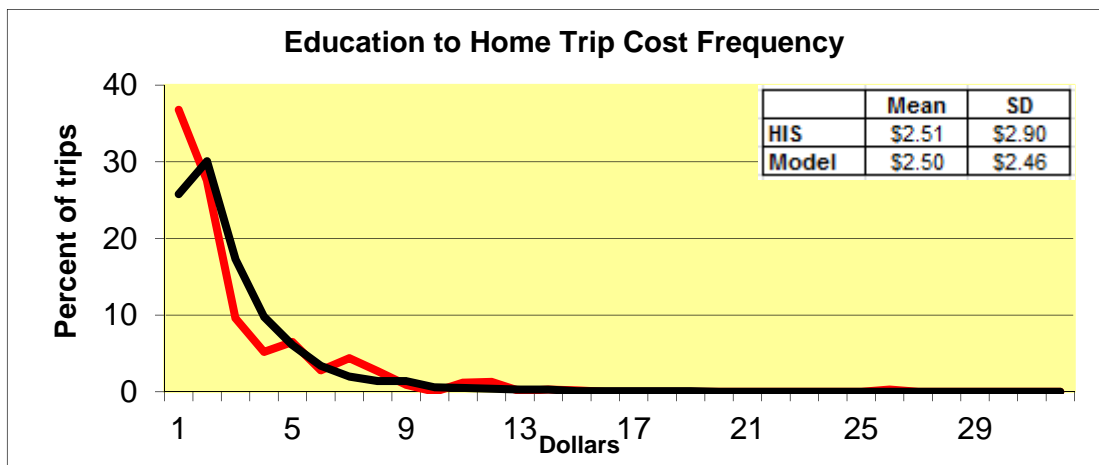
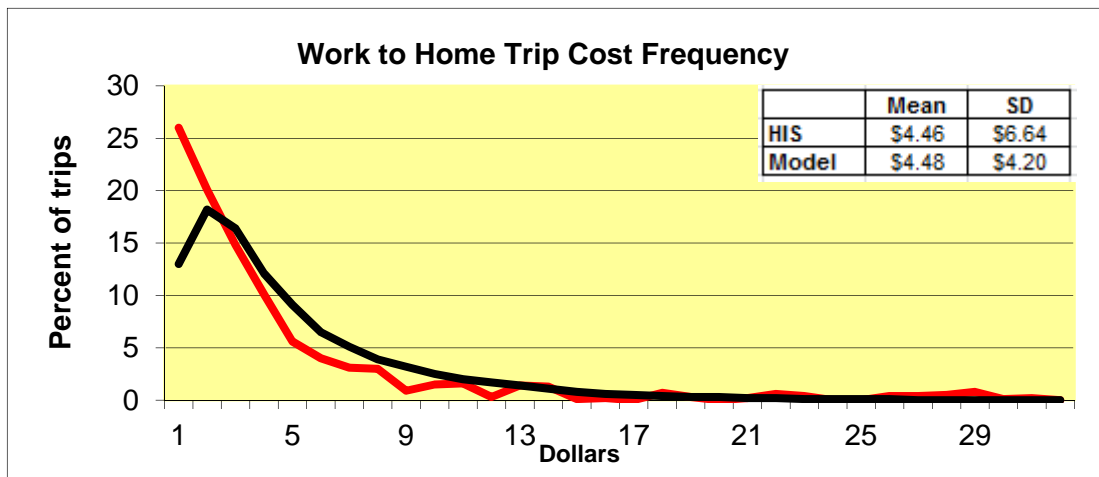
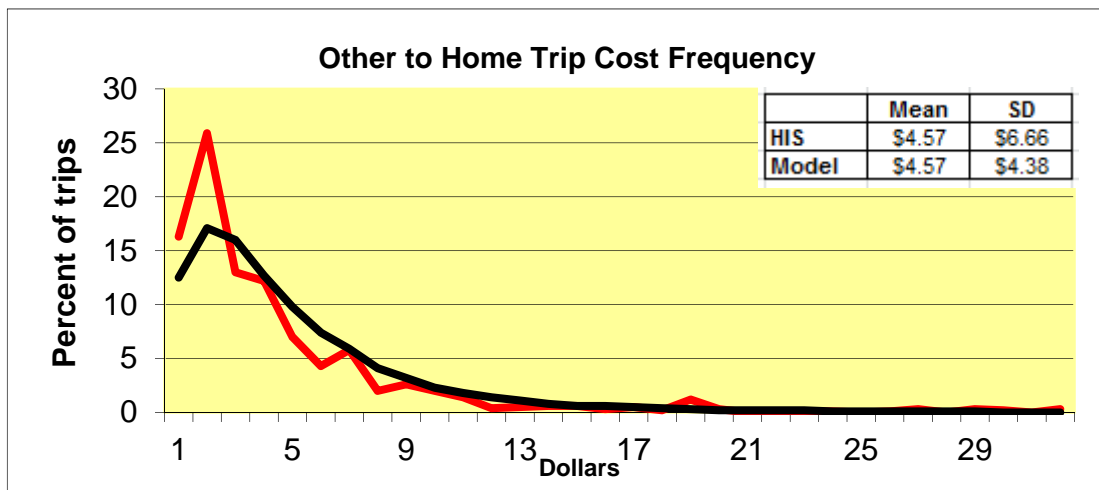




Trip Cost Frequency Plots (HTS/HTO/NHB)  
 Inter Peak Person Trips



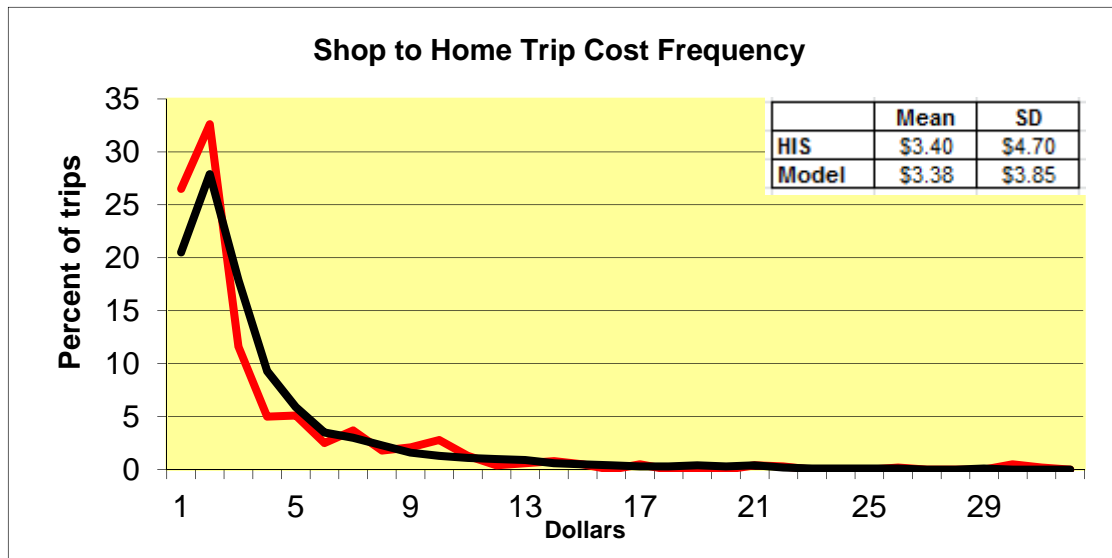
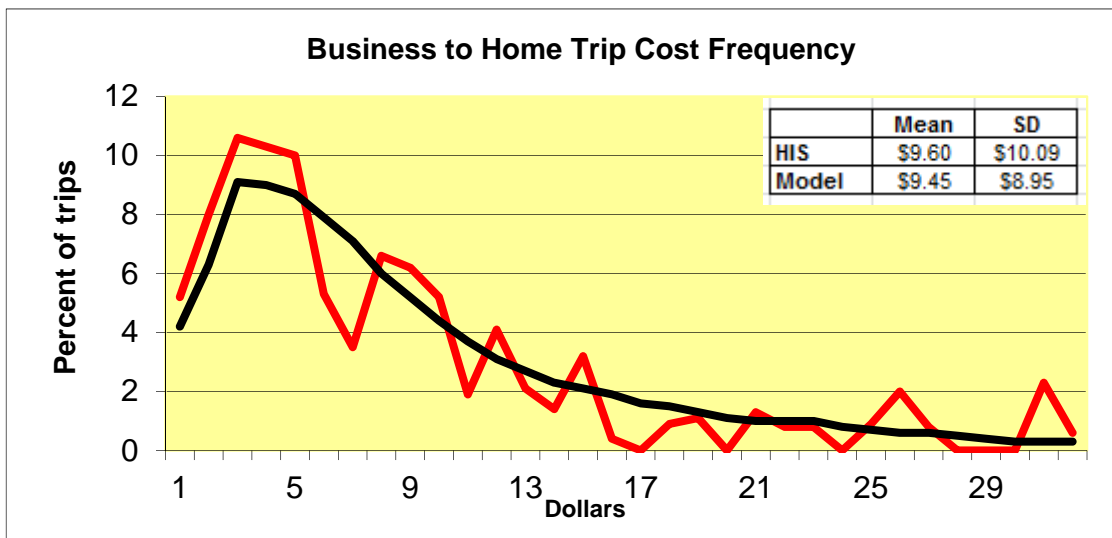
4b



Trip Cost Frequency Plots (OTH/WTH/ETH)  
 Inter Peak Person Trips



4c



Trip Cost Frequency Plots (BTH/STH)  
 Inter Peak Person Trips



4d

## 5. Comparison of 3 Step and 4 Step Modelled Trip Lengths

**Table 8** shows a comparison of the 3 and 4 step modelled trip times for each trip purpose. It highlights whether the 4-step model trips are less than the 3-step model trips.

Period	Purpose	3-step Trip Cost		4-step Trip Cost		
		Model		Model		
		Mean	SD	Mean	SD	4-step < 3-step ?
Morning Peak	Home to Work	4.37	4.10	4.22	3.96	yes
	Home to Education	4.00	3.98	2.41	2.28	yes
	Home to Business	8.46	7.82	7.94	8.58	yes
	Home to Shop	3.63	3.81	3.37	3.57	yes
	Home to Social/Rec	-	-	-	-	-
	Home to Other	4.53	4.45	4.15	3.64	yes
	Non Home Based	5.53	5.79	5.85	5.56	<b>NO</b>
	Other to Home	2.97	2.91	2.70	2.37	yes
	Work to Home	-	-	-	-	-
	Education to Home	-	-	-	-	-
	Business to Home	-	-	-	-	-
	Shop to Home	-	-	-	-	-
	Soc/Rec to Home	-	-	-	-	-
Interpeak	Home to Work	3.36	3.39	4.07	4.09	<b>NO</b>
	Home to Education	-	-	2.79	3.72	-
	Home to Business	11.41	11.03	10.22	10.80	<b>yes</b>
	Home to Shop	3.42	3.58	4.16	4.65	<b>NO</b>
	Home to Social/Rec	-	-	-	-	-
	Home to Other	6.55	5.97	5.27	4.97	<b>yes</b>
	Non Home Based	6.44	6.70	5.56	5.35	yes
	Other to Home	4.35	3.94	4.57	4.38	<b>NO</b>
	Work to Home	4.02	3.73	4.48	4.20	<b>NO</b>
	Education to Home	-	-	2.50	2.46	-
	Business to Home	6.06	8.58	9.45	8.95	<b>NO</b>
	Shop to Home	3.13	3.37	3.38	3.85	<b>NO</b>
	Soc/Rec to Home	-	-	-	-	-

**Table 8: Modelled Time Comparisons Between 3 and 4 Step**

In most instances the 4-step model trips are shorter than the 3-step model trips because of the inclusion of walking and cycling trips. There are, however examples of trip lengths slightly greater in the 4 step model, particularly in the inter peak period.

For most of the purposes where the 4-step model is higher than the 3-step model there is very little difference between the 3-step and 4-step averages, and in all cases the standard deviation of all trip lengths are very large.